Visualizing To Support the Learning of Math Facts

Moving from Mathematics Instruction to Numeracy Development

Central Alberta Regional Consortium

Alberta Regional Consortia

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“Just as phonemic awareness is a prerequisite to learning phonics and becoming a successful reader, developing number sense is a prerequisite for succeeding in mathematics.”

- David A. Sousa, *How the Brain Learns Mathematics*, 2008

What is number sense?

> Number sense is an intuition about numbers. Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents.

- Alberta Education

What is number sense?

> A true sense of number includes and goes beyond the skills of counting, memorizing facts and the situational rote use of algorithms.

- Alberta Education

What is number sense?

> Mastery of number facts occurs when students understand and recall facts and is expected to be attained by students as they develop their number sense.
>
> This mastery allows for application of number facts and facility with more complex computations.

- Alberta Education

What is number sense?

> Students require number sense in order to be able to add and subtract.

- Alberta Education

Many birds display a sense of numerosity in the number of times they repeat a particular note in their song.

Even members of the same species will learn the number of repetitions common to their location, which may be six repetitions in one woodland and seven in another.

- David A. Sousa, *How the Brain Learns Mathematics*, 2008

The ability of animals to compare the numbers of objects in collections has an obvious survival advantage.

It would help a group of animals to know whether to defend their territory if the defenders outnumber the attackers, or to retreat if there are more attackers.

- David A. Sousa, *How the Brain Learns Mathematics*, 2008

Counting:
A precursor to adding and subtracting

Students need to understand:

- **ordinality** - *sequence of numbers when counting always remains the same*
- **1-1 correspondence** - *counting*
- **cardinality** - *how many?*
- **constancy** – certain properties do not change when outside conditions change

Other precursors to adding and subtracting

Students need to know how to:

- ❑ subitize
- ❑ recognize patterns to 9

So where does visualization fit in?

> Visualization involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.

- Alberta Education Mathematics Program of Studies

Visual images and visual reasoning are important components of **number**, **spatial** and **measurement** sense.

Number visualization occurs when students create **mental representations** of numbers.

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Being able to create, interpret and describe a visual representation is part of **spatial sense** and **spatial reasoning**.

Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

- Alberta Education Mathematics Program of Studies

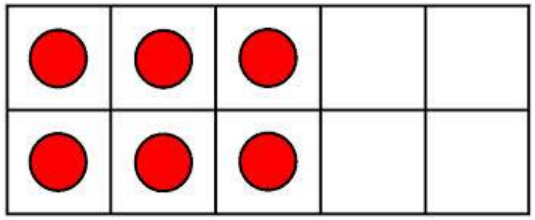
Measurement visualization goes beyond the acquisition of specific measurement skills.

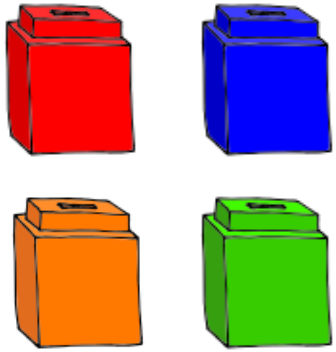
Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use.

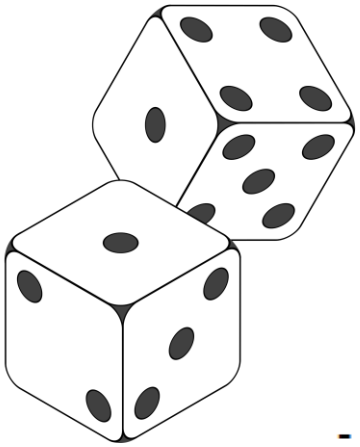
- Alberta Education Mathematics Program of Studies

How can we develop visualization?

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

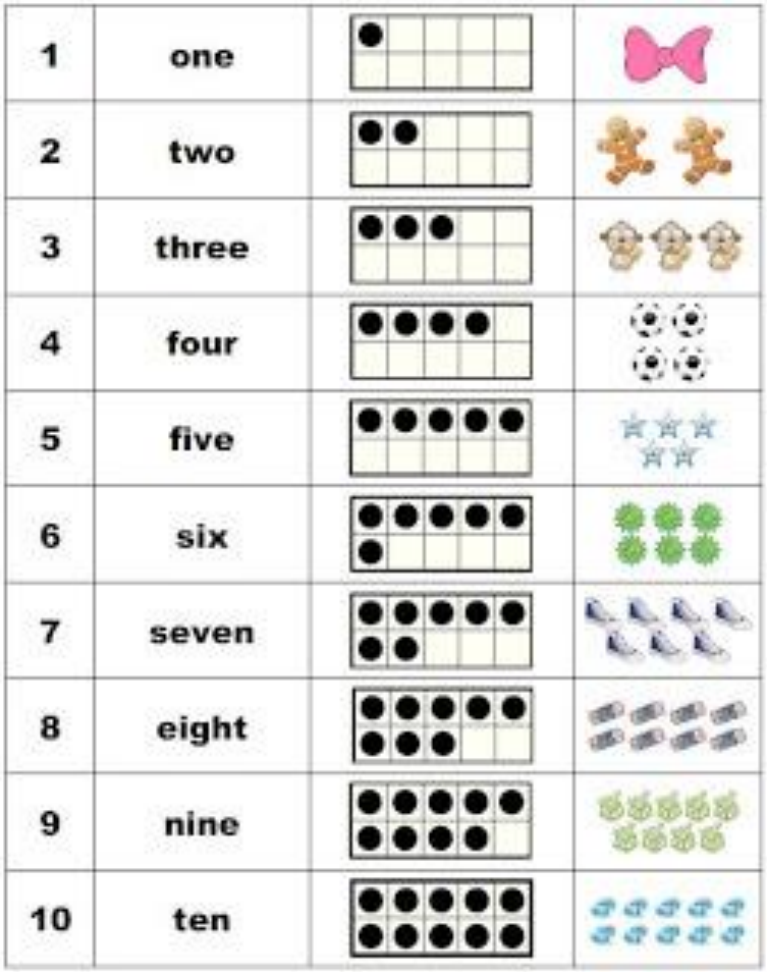






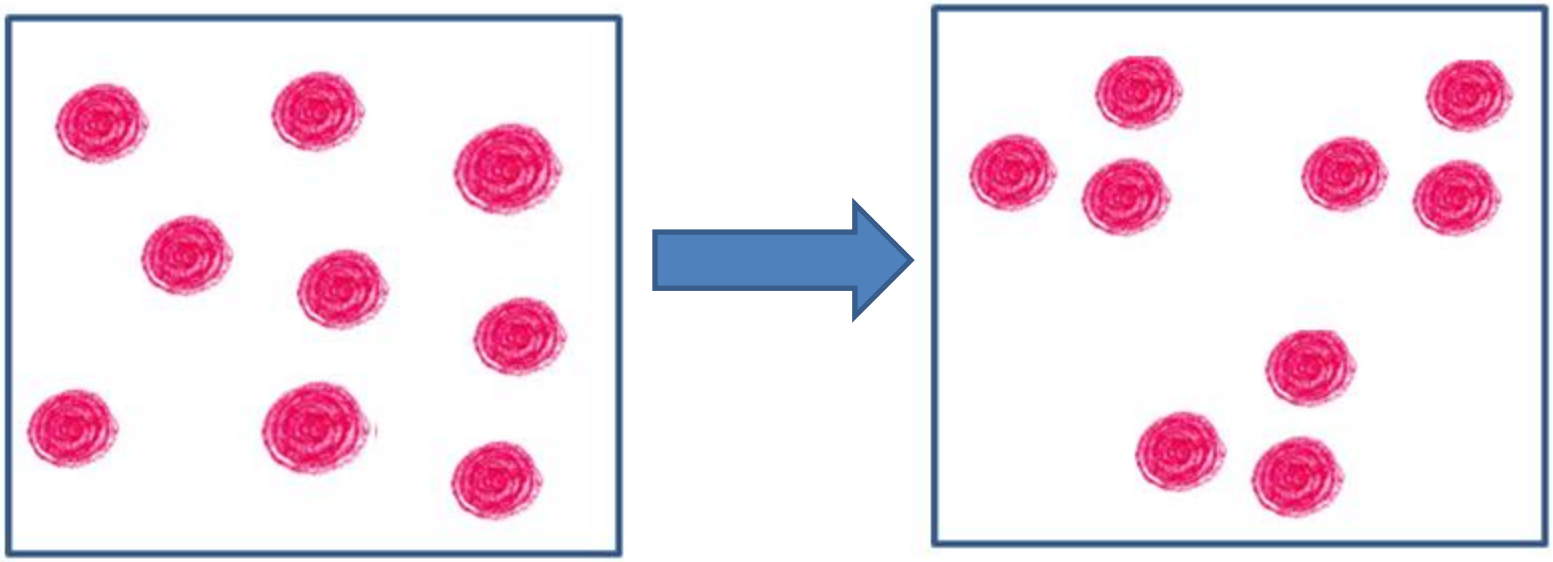
- Alberta Education Mathematics Program of Studies

“The human brain comprehends numerals as quantities, not as words. This reflex action is deeply rooted in our brains and results in an immediate attribution of meaning to numbers.”



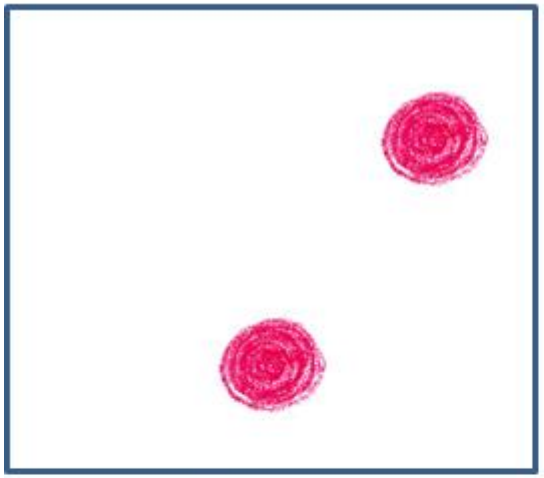
- David A. Sousa, *How the Brain Learns Mathematics*, 2008

Grouping objects can help to quickly identify larger numbers of objects.

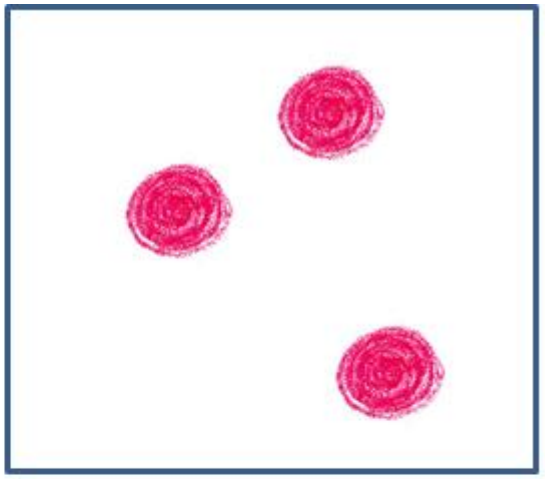


Subitizing

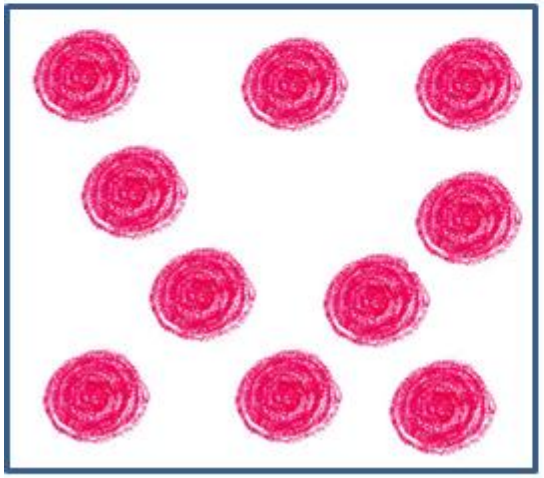
Subitizing is the ability to tell how many instantly, without counting. It is immediately recognizing an arrangement or configuration of a certain number of items.



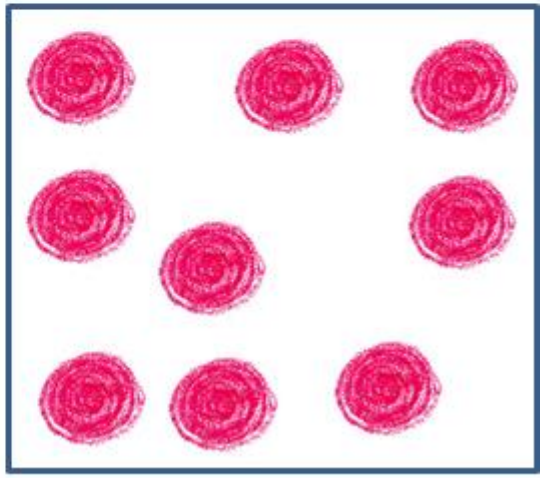
Box A



Box B

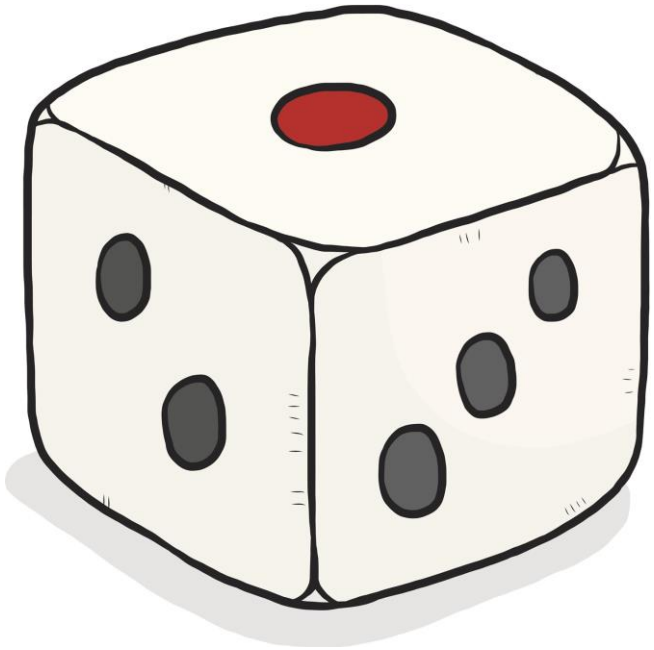


Box C



Box D

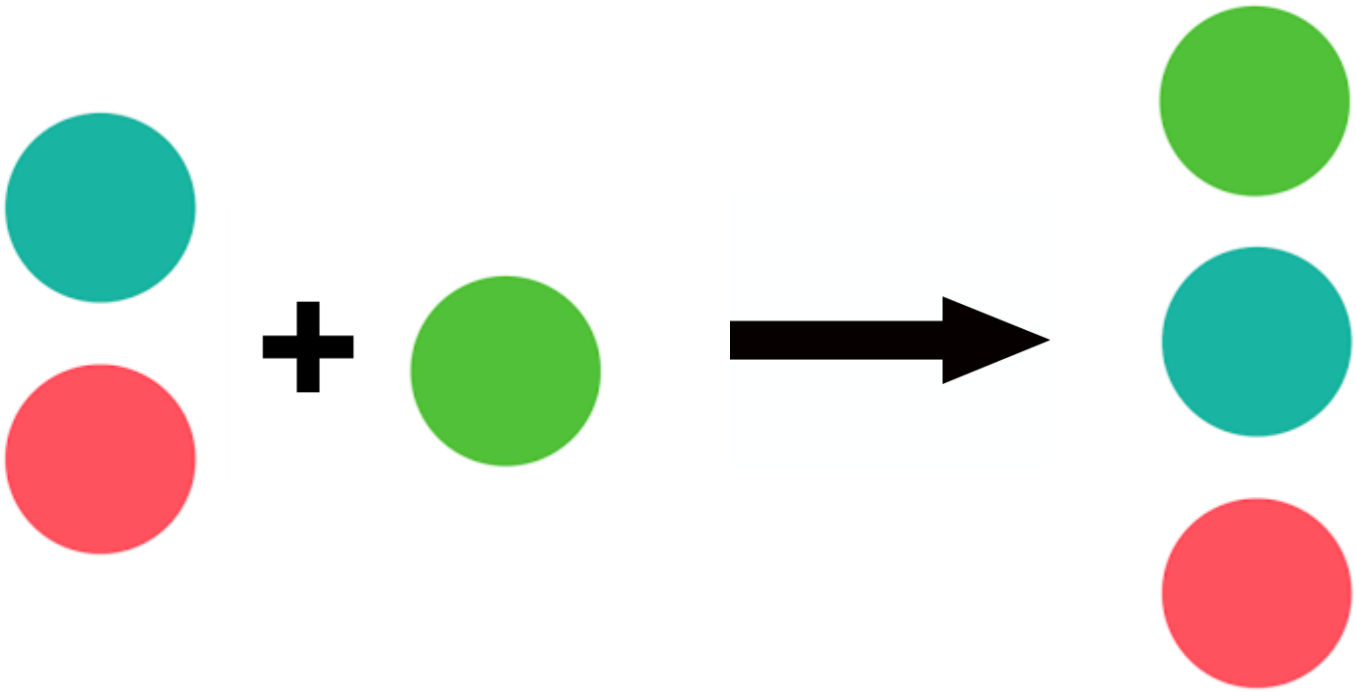
Perceptual subitizing involves recognizing a number without using other mathematical processes.



“**Perceptual subitizing** also helps children separate collections of objects into single units and connect each unit with only one number word, thus developing the process of counting.”

- David A. Sousa, *How the Brain Learns Mathematics*, 2008, p. 15

Conceptual subitizing is recognizing smaller groups and adding them together, such as two dots plus one dot equals three dots.



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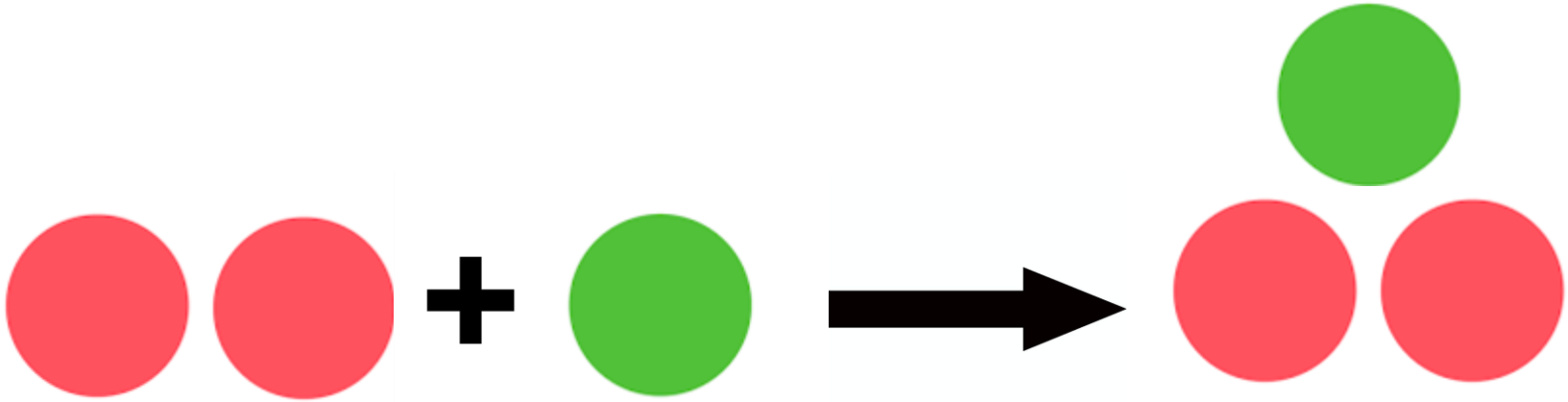
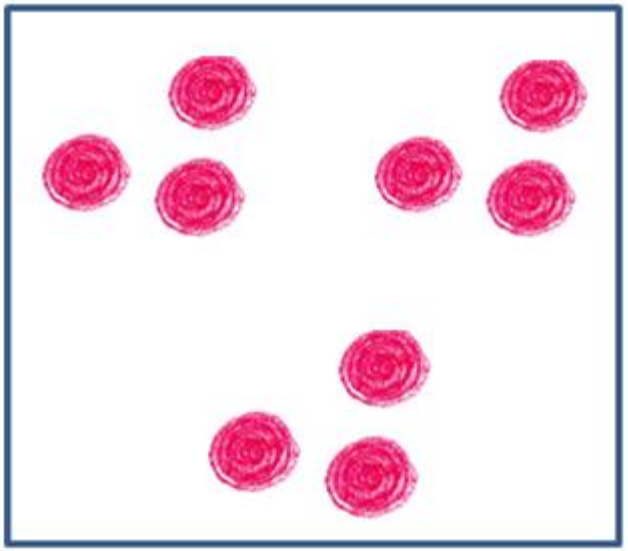


“Creating and using **conceptual subitizing** patterns help young children develop the abstract number and arithmetic strategies they will need to master counting.”

- David A. Sousa, *How the Brain Learns Mathematics*, 2008, p. 15

What Factors Make Conceptual Subitizing Easy or Hard?

The spatial arrangement of sets influences how difficult they are to subitize.



Children usually find **rectangular arrangements** easiest, followed by linear, circular, and scrambled arrangements.

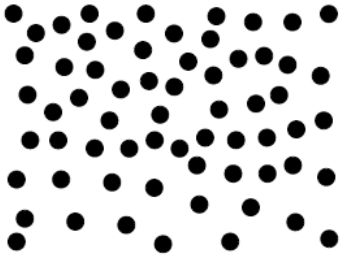
(Beck-with and Restle 1966; Wang, Resnick, and Boozer 1971).

http://www.garlandisdschools.net/uploaded/elementary_schools/parkcrest/documents/Subs.pdf

What Factors Make Conceptual Subitizing Easy or Hard?

If the arrangement does not lend itself to grouping, people of any age have more difficulty with larger sets.

(Brownell 1928)



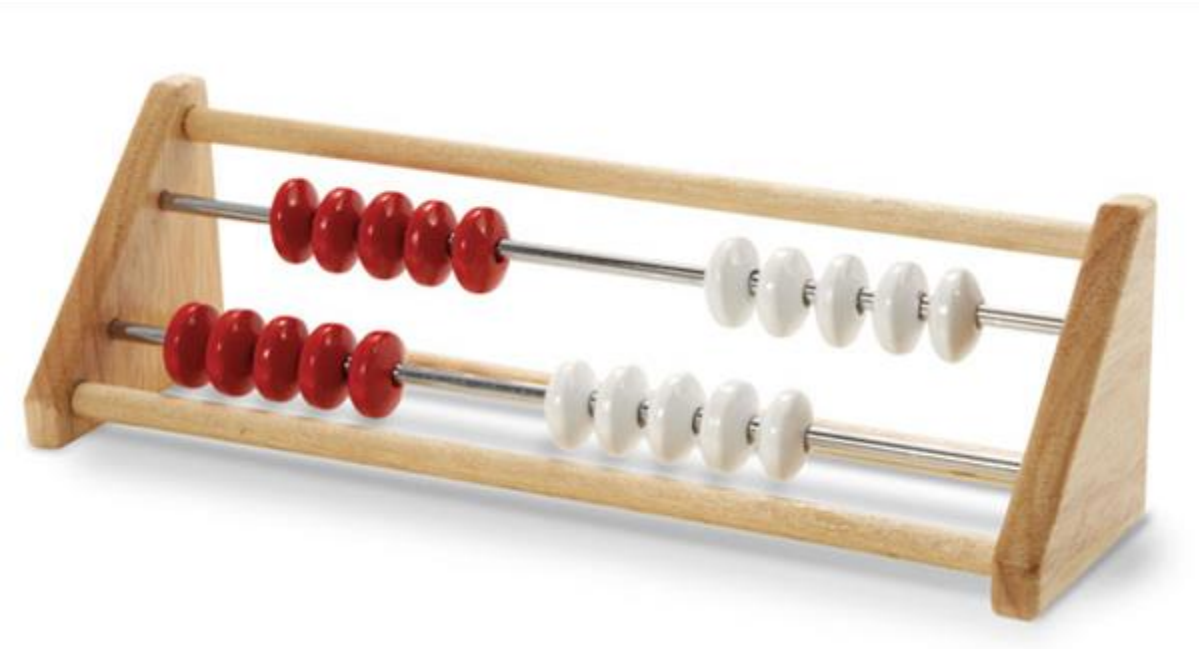
They also take more time with larger sets.

(Beckwith and Restle 1966)

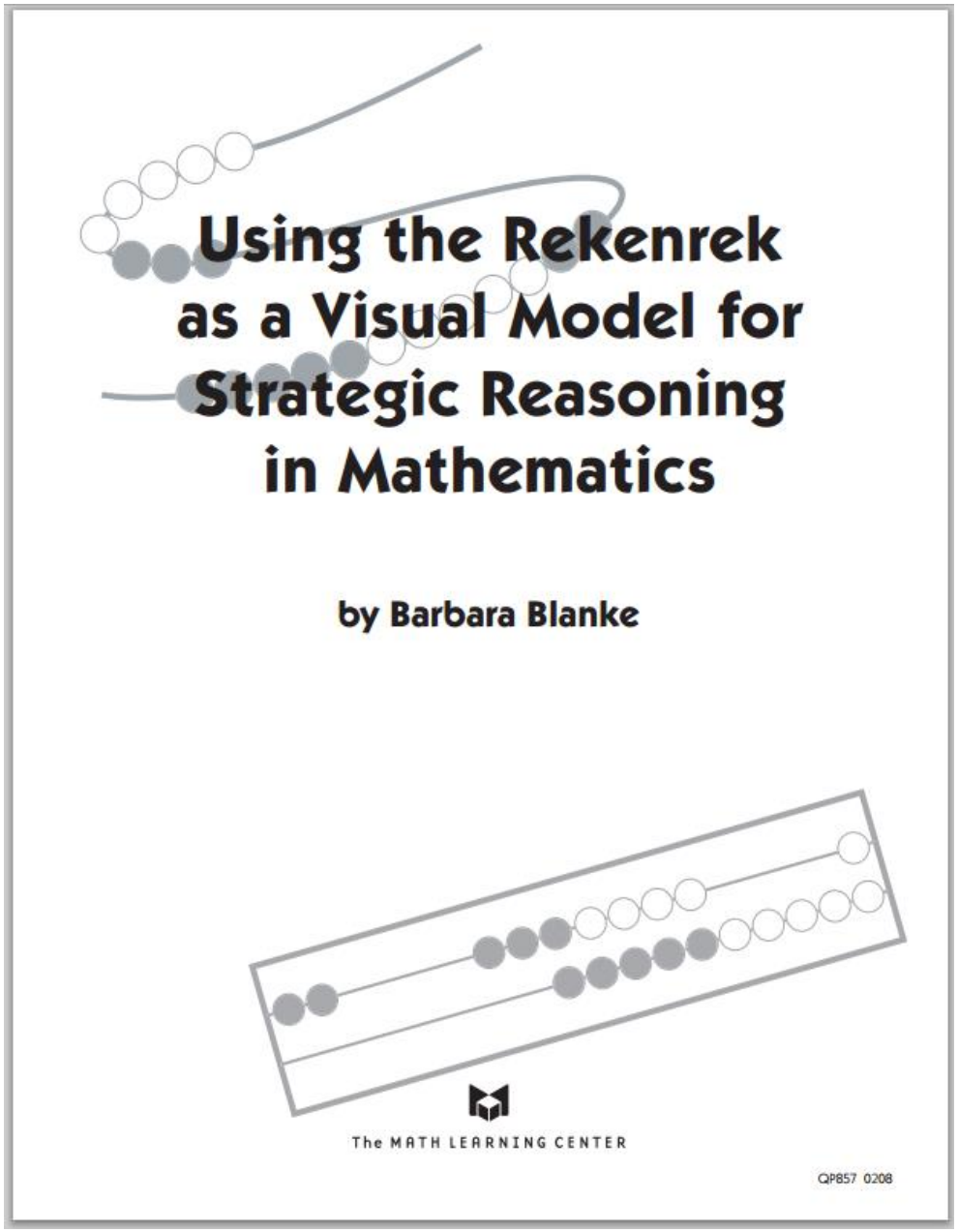
http://www.garlandisdschools.net/uploaded/elementary_schools/parkcrest/documents/Subs.pdf

How do subitizing and patterning support the development of math facts?

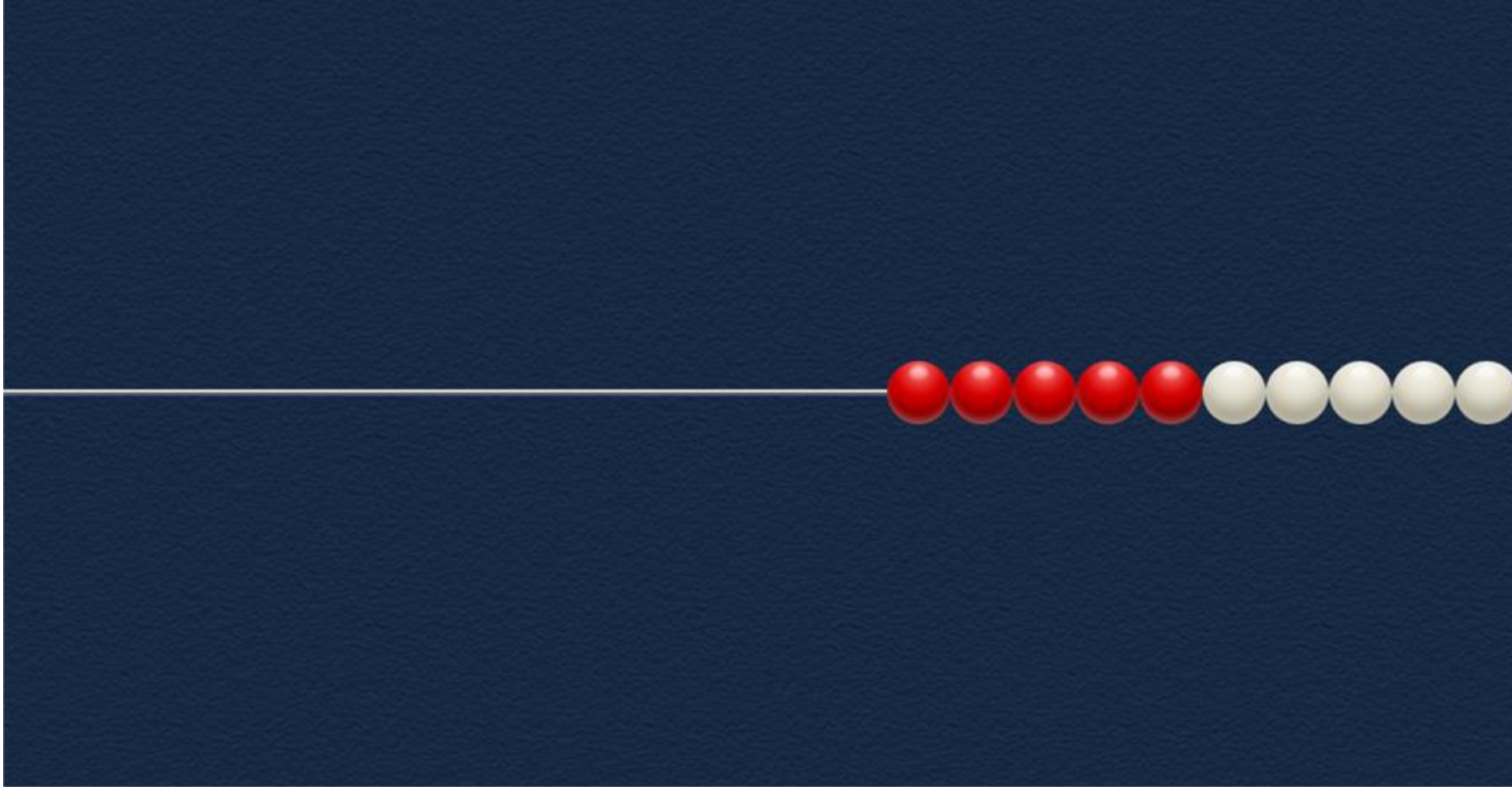
Rekenreks



http://www.edugains.ca/resources/LearningMaterials/ManipulativesSupport/TipSheets/Maniplatives_Rekenreks.pdf



http://bridges1.mathlearningcenter.org/media/Rekenrek_0308.pdf

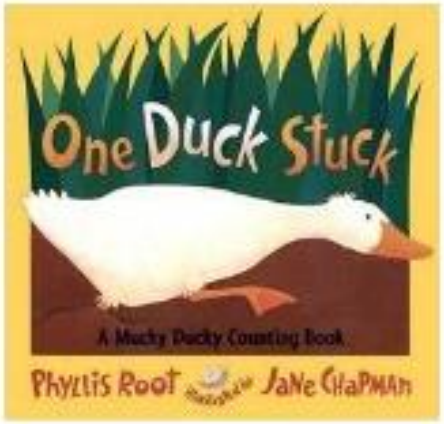


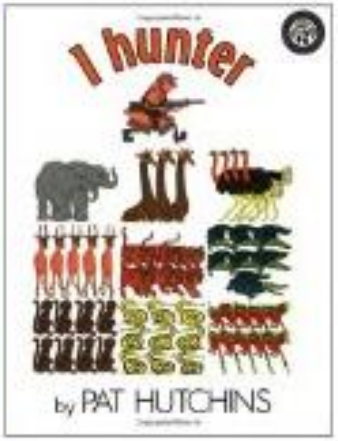
Interactive Rekenrek

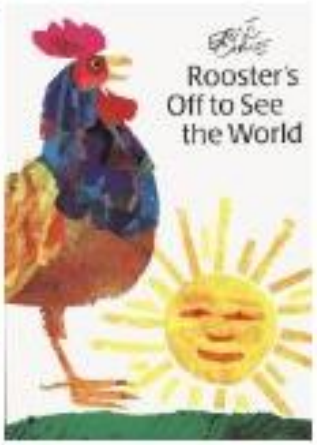
http://www.mathlearningcenter.org/web-apps/number-rack

Remember…

Literature provides numerous opportunities for students to become engaged in a mathematics concept being taught, creating a meaningful context for the student.







K-5 Math Teaching Resources

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Number

Kindergarten

1st Grade

2nd Grade

3rd Grade

4th Grade

5th Grade

Place Value Strategies

Geometry

K–2

3–5

Measurement/Data

K–2

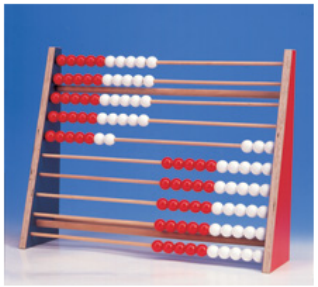
3–5

BookStore

Rekenrek Activities

The rekenrek, or arithmetic rack, was designed by Adrian Treffers, a mathematics curriculum researcher at the Freudenthal Institute in Holland, to support the natural development of number sense in children. Smaller versions consist of two rows of 10 beads. Larger versions with ten rows of ten beads are also available. Each row is made of five red beads and five white beads. This allows students to make mental images of numbers. Using 5 and 10 as anchors for counting, adding and subtracting is obviously more efficient than one-by-one counting. This tool provides learners with the visual models they need to discover number relationships and develop a variety of addition and subtraction strategies, including doubles plus or minus one, making tens, and compensation, thereby leading to automaticity of basic facts.

Possible Activities

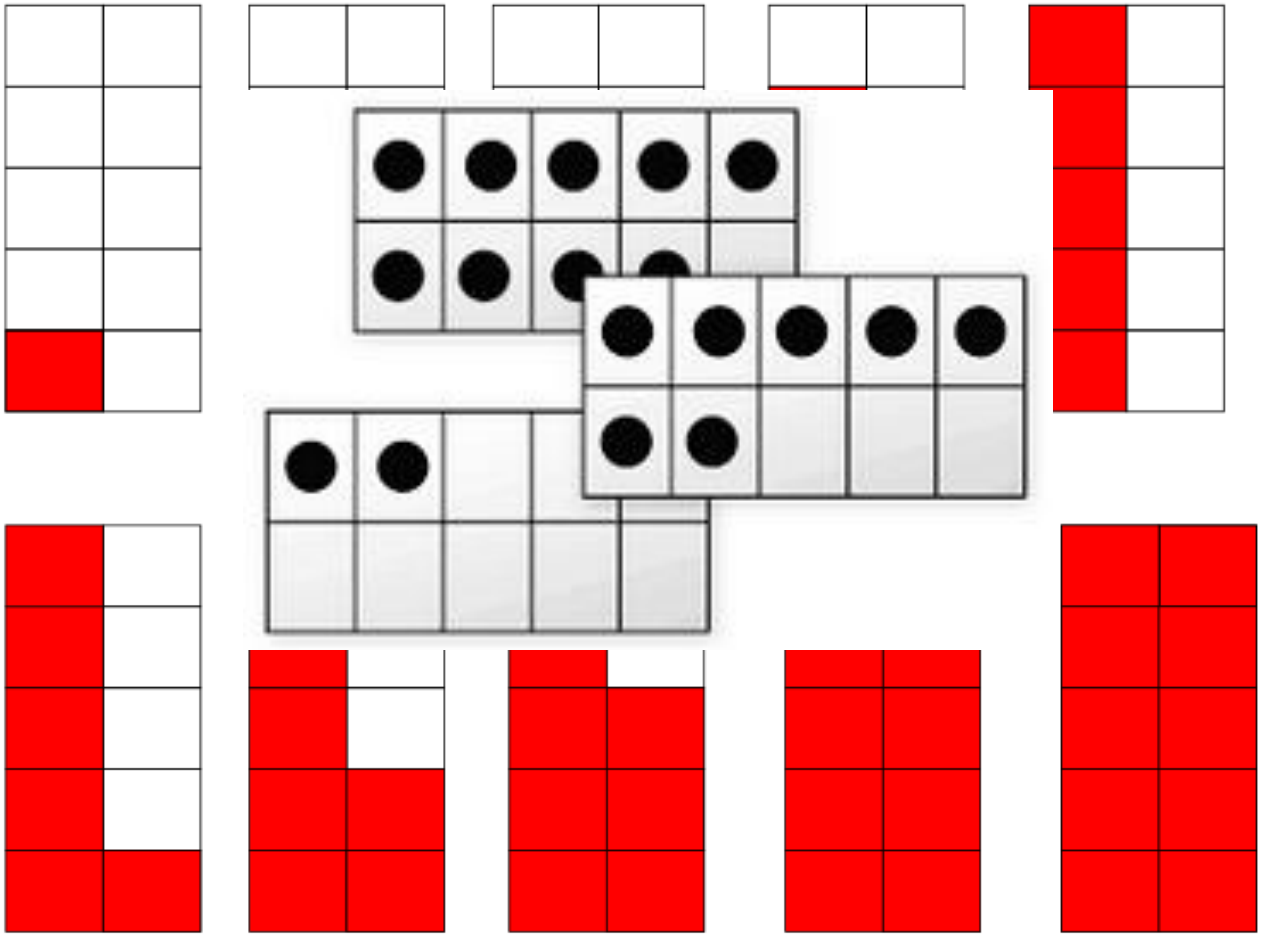


- **Meet the Rekenrek**: Begin by asking children what they notice about the rekenrek. Then introduce the 'start position'(all beads over to the far right) and have them practice sliding beads in groups rather than one by one "Put your beads in start position. Now, without touching the beads, count the first three beads in your mind. On the count of three, slide all three beads at once across the string. One… two…three!" Repeat with other numbers.
- **Show me 0–10**: Say a number, or hold up a numeral card (0–10). Ask students to show the given number by moving the beads with one push.
- **Show me 11–20**: As above but ask students to show the given number of beads using only two pushes.

http://www.k-5mathteachingresources.com/Rekenrek.html

VIDEO

Power of Ten



http://poweroften.ca/

Subitizing – learning to identify numbers without counting – is **crucial** to the development of number sense and basic fact acquisition.

Teach students to subitize as they play!

POWER OF TEN

HOME STORE GAMES+ TEACHING TOOLKIT+ WORKSHOPS+

Instructions

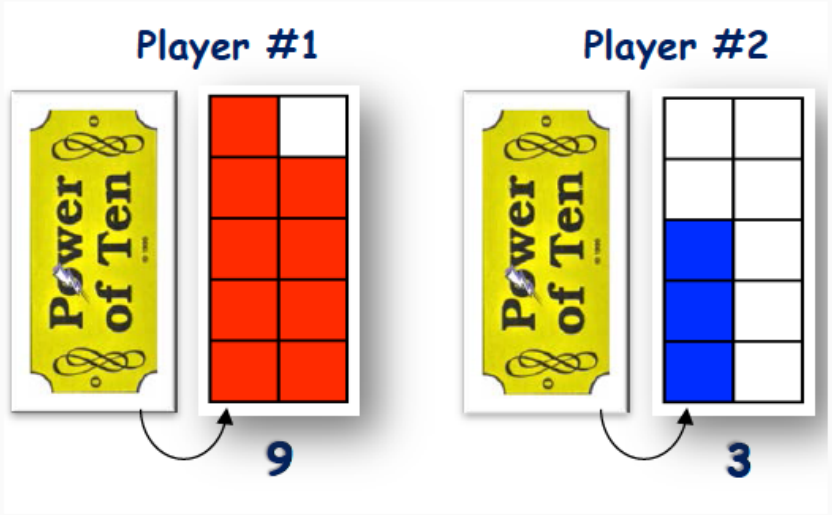
1. Each player has a deck of 20 cards. Each deck is shuffled and placed face down.

2. Player 1 and 2 simultaneously flip over the cards, and state the values of each. The player with the card showing the greatest number of coloured squares (the largest number) wins that round.

3. The Player who wins each round takes the cards and puts them in his/her own stash of winnings.

4. Very young children may be able to play this game without being able to say the numbers on the cards. However, by age 5 or 6, it's important for the child to be able to say the number out loud.

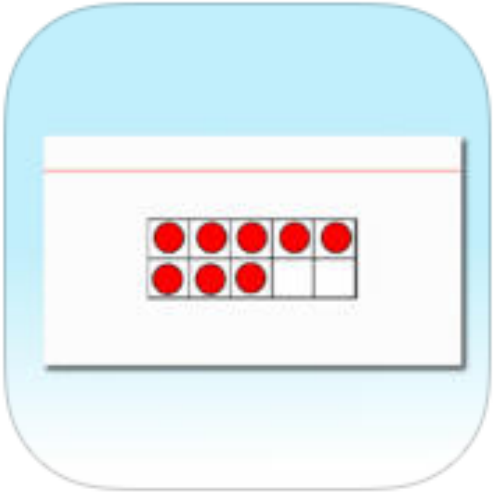
5. The winner is the person who has the most cards at the end of the game.



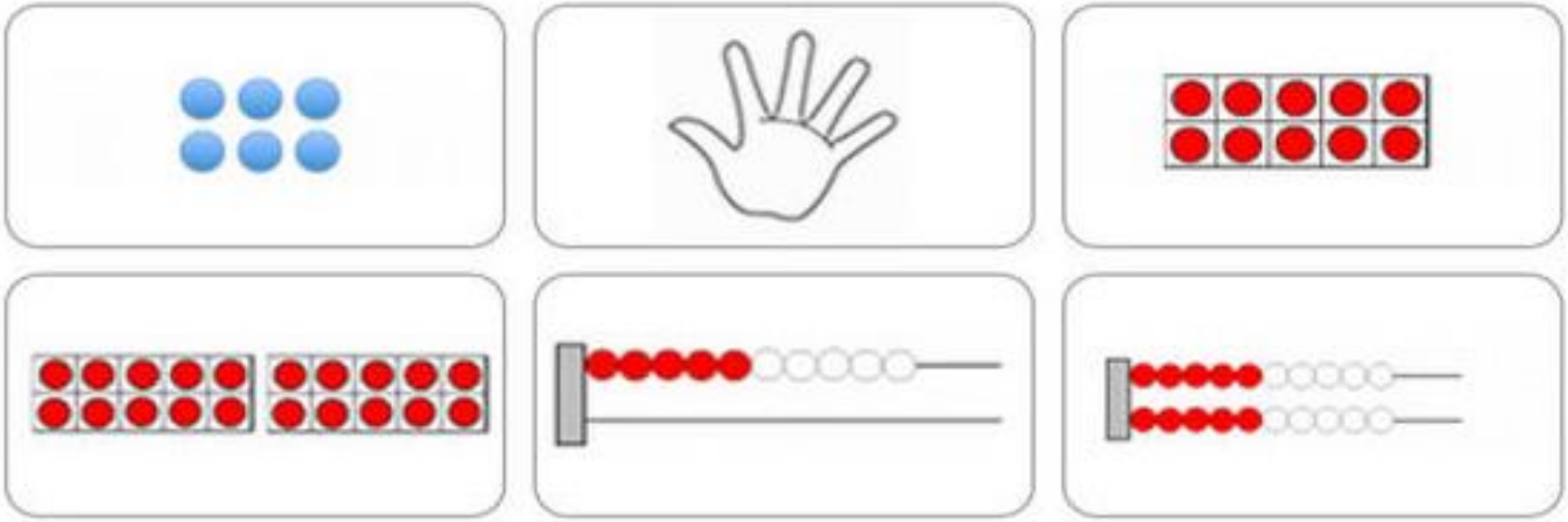
Learning through games!

ADDITION, GAMES

http://poweroften.ca/category/addition

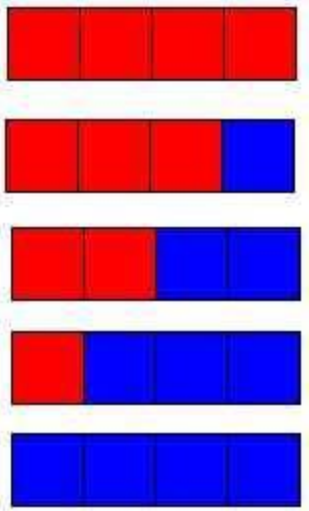


iPad - Quick Images



$0.99

Unifix Cubes



Visualization is…

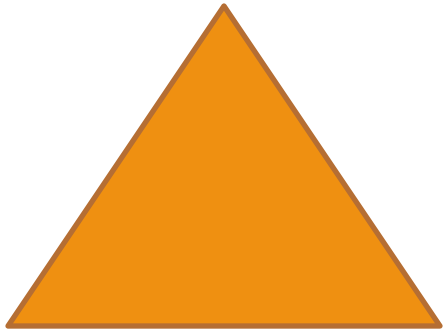
Reflecting on your learning…

Visualization is important because…

Reflecting on your learning…

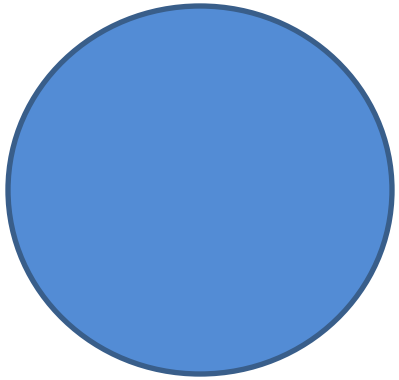
What are you already doing?

Reflecting on your learning...



What are *three important points* that you will take away?

What is something that *squared* or agreed with your thinking?



What is something still *circling* in your head?

Other resources to consider

How the Brain Learns Mathematics, David A. Sousa, Corwin Press, 2008.

Mastering the Basic Math Facts in Addition and Subtraction
http://www.heinemann.com/shared/onlineresources/e02963/oconelladd.pdf

Mastering the Basic Math Facts in Multiplication and Division
http://www.heinemann.com/shared/onlineresources/e02962/oconnellmultweb.pdf

Subitizing and Quick Images
https://sites.google.com/a/billingsschools.org/bpsccmath/other-resources/quick-images--subitizing-resources

THANK YOU

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